New finite dimensional observer for the boundary control of fluid flows

Jean-Marie Buchot, Laleh Ravanbod and Jean-Pierre Raymond *†

We present a new finite dimension estimator for infinite dimensional systems with discrete spectrum and finitely many eigenvalues in $Re(s) > -\delta$ for all $\delta > 0$ [1].
The existence of a finite dimensional compensator for stabilizing such systems was first proved by Schumacher [2, 3]. The considered class included parabolic systems with bounded control and observation operators. Subsequently, Curtain in [4–6] proposed an alternative scheme applicable to a general class of parabolic or hyperbolic systems with Neumann or Dirichlet boundary conditions and bounded or unbounded control and observation operators. It was named integral dynamic output feedback control and it was based on the relocation of the eigenvalues. All these approaches share the common disadvantage that the dynamics corresponding to an infinite number of eigenvalues in $Re(s) < -\delta$ are neglected in the estimator equations.
The approach proposed by Fujii in [7] seems to overcome this problem. There, a functional observer of Luenberger type was presented. The solution of linear parabolic initial boundary value problem was decomposed into the solution when the control input was zero and the solution when the initial condition was zero. The observation law, then, included the convolution integral of these decomposed solutions with the solution of the state feedback control law. However, the resulting observer was of infinite dimension.
Here, we propose and analyse a new observer of finite dimension, coupled with a finite dimensional feedback controller. As in [2–6], we still express the dynamics in terms of a stable and an unstable part. In our new approach, we take advantage of the fact that the stable part can be decomposed as in [8]. This is the new trick which enables us to determine our observer.

Our numerical test concerns the stabilization of two-dimensional linearized Navier-Stokes equations by a boundary control and using boundary observations of velocity and of pressure in the case of a flow around a circular cylinder. We show also that the new estimator remains efficient above an acceptable Signal to Noise Ratio for the nonlinear Navier-Stokes equations.

---

*The authors are with Université Paul Sabatier, Institut de Mathématiques,UMR CNRS 5219,31062 Toulouse Cedex 9,France. jean-marie.buchot@math.univ-toulouse.fr, laleh.hosseini@math.univ-toulouse.fr, raymond@math.univ-toulouse.fr

†The second author is supported by the FNRAE-project ECOSEA (Estimation COntrol and Stabilization of Aerodynamic flows). The other authors are partially supported by the ANR-project CORMORED BLAN-08-0115-02 and the FNRAE-project ECOSEA.

References

[1] J.P. Raymond et al., *Estimation of the velocity of a fluid flow from boundary pressure measurements*. Control and Optimization of PDEs, Graz, October 10-14, 2011.

[2] J.M. Schumacher, *A direct approach to compensator design for distributed parameter systems*.SIAM J. Control and Optimization.

[3] J.M. Schumacher, *Dynamic feedback in finite and infinite dimensional linear systems*. Mathematical Center Tracts, No. 143,Mathematisch Centrum, Amsterdam 1981.

[4] R.F. Curtain, *Finite-dimensional compensators design for parabolic distributed systems with point sensor and boundary input*. IEEE Trans. Automat. Control, AC-26, pp 98 - 104, 1982.

[5] R.F. Curtain, *Finite-dimensional compensators for some hyperbolic systems with boundary control*. Workshop on Control Theory for Distributed Parameters Systems, Vorau, Austria, July 1982.

[6] R.F. Curtain, *Stabilization of boundary control distributed systems via integral dynamic output feedback of a finite-dimensional compensator*. 5th International Conference on Analysis and Optimization of Systems, INRIA, Versaille, France, December 1982.

[7] N. Fujii, *Feedback stabilization of distributed parameters systems by a functional observer*. SIAM J. Control and Optimization,vol. 18, No. 2, pp. 108 - 120, 1980.

[8] J.P. Raymond and L. Thevenet, *Boundary feedback stabilization of the two dimensional Navier-Stokes equations with finite dimensional controllers*. Discrete and Continuous Dynamical Systems (DCDS-A), vol. 27, No. 3, pp. 1159 - 1187, 2010.