BLIND IDENTIFICATION AND SEPARATION METHODS FOR LINEAR-QUADRATIC MIXTURES AND/OR LINEARLY INDEPENDENT NON-STATIONARY SIGNALS

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ABSTRACT

This paper concerns blind mixture identification (BMI) and blind source separation (BSS). We consider non-stationary stochastic sources, more specifically sources with slight time-domain sparsity. We first propose a correlation-based BMI/BSS method for Linear-Quadratic mixtures, called LQ-TEMPCORR. We also investigate the applicability of this type of method to possibly statistically dependent (e.g. correlated) but linearly independent signals. We thus extend the scope of our linear instantaneous method LI-TEMPCORR as a spin-off of this new investigation.

1. INTRODUCTION

Many blind mixture identification (BMI) and blind source separation (BSS) methods have been reported in the literature during the last two decades. Most of them concern linear instantaneous (LI) mixtures and assume the sources to be stochastic, stationary and statistically independent, thus leading to Independent Component Analysis (ICA) approaches. Several LI-BSS methods have also been introduced in order to request less restrictive source properties than statistical independence, at the expense of additional constraints. They especially assume uncorrelated sources, which are e.g. non-stationary in addition. A specific form of non-stationarity is achieved by sources which are sparse in the time domain, i.e. sources whose variances are zero at some times and non-zero at others. This class of approaches e.g. includes our LI-TEMPCORR method [1],[2]. As for nonlinear mixtures, a few methods have been proposed in the literature, only for statistically independent sources to our knowledge.

In this paper, we address configurations which are beyond the above-defined cases. This includes two directions, where we use the above-defined type of source non-stationarity, i.e. time-domain sparsity. We mainly develop methods for linear-quadratic (LQ) mixtures, which have been considered in a few papers (see e.g. [3] and references therein). We also aim at decreasing the constraints on source dependence, especially in order to only request linearly independent signals. We investigate in which aspects of the proposed BMI and BSS methods we can combine the above two directions.

2. LINEAR-QUADRATIC MIXTURE MODEL

In this paper, we consider the following configuration. The available P signals $x_i(t)$ are mixtures of N source signals $s_j(t)$, with $P = N(N+1)/2$ in the most general case as explained below. The source signals are unknown, stochastic and real-valued. The mixing model consists of linear terms, proportional to $s_j(t)$, and quadratic cross-terms, proportional to $\tilde{s}_{jk}(t) = s_j(t)s_k(t)$. Each observed signal then reads

$$x_i(t) = \sum_{j=1,\dots,N} a_{ij}s_j(t) + \sum_{1\leq j<k\leq N} q_{ijk}\tilde{s}_{jk}(t), \quad \forall i = 1,\dots,P \qquad (1)$$

where a_{ij} and q_{ijk} are resp. linear and quadratic unknown real-valued mixing coefficients. This yields in matrix form

$$x(t) = As(t) + Q\tilde{s}(t) \qquad (2)$$

with $x(t) = [x_1(t),\dots,x_P(t)]^T$, $s(t) = [s_1(t),\dots,s_N(t)]^T$, where ^{T} denotes transposition. The column vector $\tilde{s}(t)$ consists of the signals $\tilde{s}_{jk}(t)$ in a given arbitrary order. $A = [a_{ij}]$ and $Q = [q_{ijk}]$, where i is the row index of Q and the columns of Q are indexed by (j,k) and arranged in the same order as the signals $\tilde{s}_{jk}(t)$ in $\tilde{s}(t)$. We also consider the centered version of the observations, i.e.

$$x'_i(t) = x_i(t) - E\{x_i(t)\} \qquad \forall i = 1,\dots,P. \qquad (3)$$

Eq. (1) and (3) then yield

$$x'_i(t) = \sum_{j=1,\dots,N} a_{ij}s'_j(t) + \sum_{1\leq j<k\leq N} q_{ijk}\tilde{s}'_{jk}(t), \quad \forall i = 1,\dots,P, \qquad (4)$$

where $s'_j(t)$ and $\tilde{s}'_{jk}(t)$ are resp. the centered versions of $s_j(t)$ and $\tilde{s}_{jk}(t)$. This yields in matrix form

$$x'(t) = As'(t) + Q\tilde{s}'(t) \qquad (5)$$

where the vectors $x'(t)$, $s'(t)$ and $\tilde{s}'(t)$ are the centered versions of those involved in (2).

3. PROPOSED BMI AND BSS METHODS

3.1. Identification of linear part of mixture

The first step of our method consists in identifiying the "linear part" of the mixing model, i.e. the matrix A, or more precisely the matrix $\underline{A} = [\underline{a}_{ij}]$, where

$$\underline{a}_{ij} = \frac{a_{i,\sigma(j)}}{a_{1,\sigma(j)}} \qquad \forall i = 1 \ldots P, \qquad \forall j = 1, \ldots, N \quad (6)$$

and $\sigma(.)$ is a permutation. $\underline{A}$ is therefore a modified version of A, where the columns are permuted and each column is rescaled with respect to the value in its first row, i.e. to its linear contribution in observation $x_1(t)$ (the columns of A could be rescaled with respect to another observation instead). This corresponds to the classical indeterminacies of the LI-BSS problem. To ensure that all parameters in (6) are defined, we set the following constraint:

Assumption 1 *All entries of the first row of A are non-zero.*

We now introduce the other assumptions that we use to identify the linear part of the mixture.

Assumption 2 *A is a full-column-rank matrix.*

Definition 1 *A signal is said to be "active" at time t if it has non-zero variance at that time*[1]*. It is said to be "inactive" at time t if it has zero variance at that time and may then be considered as a deterministic constant.*

Definition 2 *A source $s_j(t)$ is said to be "isolated" at time t if only this source, among all sources $s_1(t), \ldots, s_N(t)$, is active at that time.*

Definition 3 *A source $s_j(t)$ is said to be "visible" in the time domain if there exist as least one time t when it is isolated.*

Assumption 3 *Each source $s_j(t)$ is visible in the time domain.*

The considered sources are therefore non-stationary[2], since their variances are zero at some times and non-zero at others. Moreover, they are only requested to have slight sparsity in the time domain, in the sense that they are allowed to overlap almost everywhere: for each source, we only request the existence of a time t (i.e. a short time window for practical estimation) when only this source is active.

Assumption 4 *For any considered time t, the signals which are contained by $s'(t)$ and $\tilde{s}'(t)$ and which are active at that time are linearly independent.*

We recall (see [4], Ed. 2002, p. 251) that the real-valued random variables w_i are linearly independent if $E\{(c_1w_1 + \cdots + c_nw_n)^2\} > 0$ for any $C \neq 0$, where $C = [c_1, \ldots, c_n]$ and $E\{.\}$ stands for expectation. It should be noted that if the active signals in $s'(t)$ and $\tilde{s}'(t)$ are uncorrelated, then Assumption 4 is met. However, there also exist cases with active signals in $s'(t)$ and $\tilde{s}'(t)$ such that Assumption 4 is still met, although these signals are partly correlated. This shows the attractiveness of our approach, even when we restrict ourselves to LI mixtures: there exist signals which cannot be separated by ICA approaches and classical second-order-statistic BSS methods because they are correlated, while our method still applies to them.

Assumption 5 *All sources $s_1(t), \ldots, s_N(t)$ are zero-mean at any time t.* [3]

The identification of $\underline{A}$ is performed by mainly taking advantage of Assumption 3, i.e. of the existence of times when each source is isolated. Such times should first be detected, so as to operate at those times. The method that we use to this end may be intuitively introduced as follows: at a time t when a source is isolated, say $s_l(t)$, the observed signals (1) become restricted to

$$x_i(t) = a_{il}s_l(t) \qquad \forall i = 1 \ldots P. \quad (7)$$

All observed signals are therefore proportional at any such time. So, an appealing approach for detecting these times consists in checking the cross-correlation coefficients between the observed signals $x_1(t)$ and $x_i(t)$, defined as

$$\rho_{x_1x_i}(t) = \frac{E\{x'_1(t)x'_i(t)\}}{\sqrt{E\{[x'_1(t)]^2\}E\{[x'_i(t)]^2\}}}. \quad (8)$$

We indeed prove in the Appendix that a necessary and sufficient condition for a source to be isolated at time t is

$$|\rho_{x_1x_i}(t)| = 1 \qquad \forall i = 2 \ldots P. \quad (9)$$

Again using correlation parameters associated to the observed signals then makes it possible to identify part of the matrix $\underline{A}$ at each time when a source is isolated. More precisely, when only $s_l(t)$ is active, (21) entails

$$\frac{E\{x'_i(t)x'_1(t)\}}{E\{[x'_1(t)]^2\}} = \frac{a_{il}}{a_{1l}} \qquad \forall i = 2 \ldots P. \quad (10)$$

The set of values thus obtained for all observations indexed by i identifies one of the columns of $\underline{A}$, as shown by (6), which also indicates that all values in the first row of $\underline{A}$ are equal to 1. By repeatedly performing such column identifications for times associated to all sources, we eventually identify the overall matrix $\underline{A}$. The details of this procedure are the same as in our previous LI-TEMPCORR method and may be found in [1],[2].

We thus succeeded in achieving the BMI task for the linear part of the mixture model (2), despite the presence of its quadratic part. We now proceed to the other aspects of the BMI and BSS tasks for this model. It should first be noted that they are straightforward in the specific case when the model (2) is restricted to a linear one, i.e. when $Q = 0$. Indeed, we then have completely identified the model, up to its indeterminacies. BSS is then achieved just by computing the signal vector $\underline{A}^\dagger x(t)$, where † denotes

[1] Each considered time area is restricted to a single time t in this theoretical statistical framework. However, in practice, all signal moments are estimated over time windows and each considered time area then consists of such a window.

[2] More precisely, they are long-term non-stationary, but they should be short-term stationary in practice in order to make it possible to estimate the above-mentioned signal moments over short time windows.

[3] The observations may then be non-zero-mean, due to the nonlinear nature of the mixture and the possible source correlation. We therefore consider the centered version $x'_i(t)$ of the observations hereafter.

the pseudo-inverse (or we compute the vector $\underline{A}^{-1}x(t)$ when using $P = N$ for LI mixtures). This vector is equal to the source vector $s(t)$, up to the scale and permutation BSS indeterminacies (and, of course, up to estimation errors). This LI-BSS method itself is nothing but the above-mentioned LI-TEMPCORR approach. However, this paper provides new results concerning its applicability: while the demonstration provided in [1],[2] only guaranteed that it applies to uncorrelated sources, we here extended that result by proving that it is relevant for a wider class of signals, i.e. linearly independent sources.

3.2. Cancellation of linear part of mixture

We then aim at deriving a set of L signals $z_l(t)$ from the observations $x_i(t)$, in such a way that these signals $z_l(t)$ only contain quadratic cross-terms, i.e. terms proportional to $\tilde{s}_{jk}(t)$. To this end, we consider signals defined as

$$z_l(t) = x_1(t) - \sum_{i=2}^{P} c_{li} x_i(t) \qquad \forall l = 1, \ldots, L. \tag{11}$$

Combining this expression with (1) and (6) yields

$$\begin{aligned} z_l(t) \quad &= \sum_{j=1,\ldots,N} a_{1,\sigma(j)} s_{\sigma(j)}(t)[1 - \sum_{i=2}^{P} \underline{a}_{ij} c_{li}] \\ &+ \sum_{1 \leq j < k \leq N} r_{ljk} \tilde{s}_{jk}(t) \qquad \forall l = 1, \ldots, L. \end{aligned} \tag{12}$$

To obtain a signal $z_l(t)$ which contains no linear terms associated to $s_j(t)$, we select the coefficients c_{li} so that

$$\sum_{i=2}^{P} \underline{a}_{ij} c_{li} = 1 \qquad \forall j = 1, \ldots, N. \tag{13}$$

For a given index l, this yields a set of N equations, where the unknowns are the $P-1$ values of c_{li}, while the (estimated) coefficients $\underline{a}_{ij}$ are available from Section 3.1. If $P - 1 = N$, this set of linear equations has a single solution, i.e. we can only create one such signal $z_l(t)$. More generally speaking, whatever $M \geq 0$, if $P-1 = N+M$, we can create $M + 1$ linearly independent signals $z_l(t)$. Besides, (12) then reduces to

$$z_l(t) = \sum_{1 \leq j < k \leq N} r_{ljk} \tilde{s}_{jk}(t) \qquad \forall l = 1, \ldots, L \tag{14}$$

i.e. these signals $z_l(t)$ are then only mixtures of the quadratic signals $\tilde{s}_{jk}(t)$. Moreover, there exist $N(N-1)/2$ signals[4] $\tilde{s}_{jk}(t)$ in the observations (1). We want the set of mixtures $z_l(t)$ of the signals $\tilde{s}_{jk}(t)$ to be invertible. We therefore set the numbers L and P of recombined signals $z_l(t)$ and observations $x_i(t)$ to $L = M+1 = N(N-1)/2$ and therefore $P = N + M + 1 = N(N+1)/2$.

So, we thus obtained the following result: by solving Eq. (13) and deriving the resulting signals according to (11), we obtain the set of LI mixtures $z_l(t)$ of the signals $\tilde{s}_{jk}(t)$ defined by (14), which is invertible when $[r_{ljk}]$ is assumed to be invertible. These mixed signals may then be used in various ways, as will now be shown.

[4]Or less if all coefficients for at least one signal $\tilde{s}_{jk}(t)$ are zero.

3.3. Remaining BMI and BSS tasks

One may then proceed in different ways, depending on which parts of the BMI and BSS tasks should be performed in the considered application and which constraints on the sources are acceptable. We now explore these alternatives.

3.3.1. A method based on non-stationarity conditions

We first again focus on methods for signals which are time-domain sparse, and therefore non-stationary. One may then process the LI mixtures $z_l(t)$ of the signals $\tilde{s}_{jk}(t)$, defined in (14), by adapting the approach of Section 3.1 to this new context. This achieves both BMI for the mixing matrix in (14) (but not yet for the original matrix Q in (2)) and BSS for the signals $\tilde{s}_{jk}(t)$ (but not yet for the signals $s_j(t)$). This adaptation of the approach of Section 3.1 requires to extend the assumptions accordingly. Especially, we then need times when a single signal $\tilde{s}_{jk}(t)$ is active, i.e. essentially times when only the two corresponding sources $s_j(t)$ and $s_k(t)$ are simultaneously active.

It should also be noted that in the basic configuration with $N = 2$ sources, only a single signal $\tilde{s}_{jk}(t)$ exists, i.e. $s_1(t)s_2(t)$. This signal is then directly provided by the method described in Section 3.2, so that the stage described in the current section then disappears.

3.3.2. A method also using other correlation parameters

The method defined in Section 3.3.1 yields scaled permuted versions of the signals $\tilde{s}_{jk}(t)$, i.e. it provides a set of signals

$$y_l(t) = \lambda_{jk} \tilde{s}_{jk}(t) \qquad \forall l = 1, \ldots, L. \tag{15}$$

We now propose a simple method which may then be applied to these signals when one also wants to identify the matrix Q and/or to separate the signals $s_j(t)$. Considering the signals which are contained by $s'(t)$ and $\tilde{s}'(t)$ at times when they are active, we request them to be uncorrelated, unlike in the previous stages of our approach. Denoting $y'_l(t)$ the centered version of $y_l(t)$, we then have if $\tilde{s}_{jk}(t)$ is active

$$\begin{aligned} \alpha_{il} \quad &= \quad \frac{E\{y'_l(t) x'_i(t)\}}{E\{[y'_l(t)]^2\}} = \frac{q_{ijk}}{\lambda_{jk}} \qquad \forall i = 1 \ldots P, \\ &\qquad \forall l = 1, \ldots, L. \end{aligned} \tag{16}$$

This may be interpreted as in Section 3.1, i.e. one may build the matrix $[\alpha_{il}]$, where each column l corresponds a one signal $\tilde{s}_{jk}(t)$. Eq. (16) then shows that this matrix is equal to Q, up to the scale and permutation indeterminacies. This completes all BMI tasks. Moreover, consider the signals

$$u_i(t) = x_i(t) - \sum_{l=1}^{L} \alpha_{il} y_l(t) \qquad \forall i = 1, \ldots, P. \tag{17}$$

Denoting $u(t)$ the column vector of signals $u_i(t)$, Eq. (1), (15), (16) and (17) then yield in matrix form

$$u(t) = As(t). \tag{18}$$

BSS is then straightforwardly achieved for the original sources $s_j(t)$ by computing the vector $\underline{A}^\dagger u(t)$.

3.3.3. A method only using variance parameters

Eventually, if one is mainly interested in the BSS of the sources $s_j(t)$, the method of Section 3.3.1 and its constraints may be avoided, again at the expense of requesting the uncorrelation of the signals which are contained by $s'(t)$ and $\tilde{s}'(t)$ (considered at times when they are active). To this end, we introduce the signals

$$v_i(t) = x_i(t) - \sum_{l=1}^{L} d_{il} z_l(t) \quad \forall i = 1, \dots, P. \quad (19)$$

It may be shown that, by adapting all coefficients d_{il} so as to minimize the variances of all signals $v_i(t)$, the vector $v(t)$ consisting of these signals becomes equal to $As(t)$. BSS is then achieved for the original sources $s_j(t)$ by computing the vector $\underline{A}^\dagger v(t)$.

4. SIMULATION RESULTS

We generated two statistically independent, uniformly distributed, 10000-sample sources. The first 1000 samples of the first source and the last 1000 samples of the second source were then replaced by zeros to create two windows where each source is resp. isolated. The two sources were then mixed using model (1) with random parameters to generate 3 mixed signals. The approach proposed in Section 3.1 was used to detect the above single-source windows and to identify the entries of $\underline{A}$. Then, the procedures described in Sections 3.2 and 3.3.2 were applied to cancel the linear part of the mixture, to identify the coefficients α_{il} in (16) and to estimate the sources. The experiment was repeated 100 times, for 100 different seed values of the random variable generator and 100 different values of the mixing model coefficients. For each experiment, the output Signal to Interference Ratio (in dB) was computed by

$$SIR = 0.5 \sum_{i=1}^{2} 10 \log_{10} \frac{E[s_i^2]}{E[(\hat{s}_i - s_i)^2]}, \quad (20)$$

after normalizing the estimated sources, $\hat{s}_i(t)$, so that they have the same variances as the source signals, $s_i(t)$. The mean and standard deviation of SIR on the 100 experiments were 46.8 dB and 7.0 dB. This proves that the proposed method achieves BMI and BSS with high accuracy.

5. CONCLUSION

In this paper, we considered two types of assumptions for stochastic signals, i.e. mainly non-stationarity/sparsity and optionally linear independence. We have developed and analyzed several versions of a BMI and BSS method based on these assumptions, both for LI and LQ mixtures. Our method for LQ mixtures may be considered as an extension of our previous LI-TEMPCORR approach and is therefore called LQ-TEMPCORR. Our future investigations will especially aim at further reducing its assumptions and extending it to more general mixture models.

A. APPENDIX

We here investigate the properties of the cross-correlation coefficients $\rho_{x_1 x_i}(t)$ defined in (8), depending on the number of active sources at the considered time t. We assume that at least one source is active, otherwise these coefficients $\rho_{x_1 x_i}(t)$ are undefined. If only one source is active, say $s_l(t)$, it may be shown that

$$E\{x_i'(t) x_k'(t)\} = a_{il} a_{kl} E\{[s_l'(t)]^2\} \quad \forall i, k = 1 \dots P. \quad (21)$$

Eq. (8) then yields

$$|\rho_{x_1 x_i}(t)| = 1 \qquad \forall i = 2 \dots P. \quad (22)$$

Conversely, assume that (22) is met. We then have (see [4], Ed. 1965, p. 210)

$$\forall i = 2 \dots P, \quad \exists\, \alpha_i,\ \beta_i,\ /\ x_i(t) = \alpha_i x_1(t) + \beta_i. \quad (23)$$

This yields for the centered version of the observations

$$\forall i = 2 \dots P, \quad \exists\, \alpha_i,\ /\ x_i'(t) = \alpha_i x_1'(t). \quad (24)$$

Eq. (4) and (24) then result in

$$\sum_{j=1,\dots,N} (a_{ij} - \alpha_i a_{1j}) s_j'(t) \quad + \quad (25)$$
$$\sum_{1 \le j < k \le N} (q_{ijk} - \alpha_i q_{1jk}) \tilde{s}_{jk}'(t) \quad = \quad 0, \forall i = 2, \dots, P.$$

Denoting $\mathcal{A}(t)$ the set of indices of active sources at the considered time t, one derives from (25) that

$$E\{[\sum_{\substack{j = 1, \dots, N \\ j \in \mathcal{A}(t)}} (a_{ij} - \alpha_i a_{1j}) s_j'(t) \quad +$$
$$\sum_{\substack{1 \le j < k \le N \\ j \in \mathcal{A}(t), k \in \mathcal{A}(t)}} (q_{ijk} - \alpha_i q_{1jk}) \tilde{s}_{jk}'(t)]^2\} \quad = \quad 0,$$
$$\forall i = 2, \dots, P. \quad (26)$$

Assumption 4 then implies for any i, with $i = 2 \dots P$, that

$$a_{ij} - \alpha_i a_{1j} \quad = \quad 0,\ \forall j = 1, \dots, N,\ j \in \mathcal{A}(t) \quad (27)$$
$$q_{ijk} - \alpha_i q_{1jk} \quad = \quad 0,\ 1 \le j < k \le N, j \in \mathcal{A}(t), k \in \mathcal{A}(t).$$

It may then be derived from (27) that, if several sources were active at time t, the corresponding columns of A would be proportional. This is in contradiction with Assumption 2. We therefore conclude that, if (22) is met, only one source is active. As an overall result, (22) is met when and only when one source is active.

B. REFERENCES

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